Department of
Economics and Finance

Working Paper No. 13-05

Economics and Finance Working Paper Series

Guglielmo Maria Caporale and Luis Alberiko Gil-Alana

Long Memory in the Ukrainian Stock Market

March 2013

http://www.brunel.ac.uk/economics

LONG MEMORY IN THE UKRANIAN STOCK MARKET

1. Introduction

This paper analyses the behaviour of stock prices in Ukraine by modelling the PFTS stock market index. Specifically, it examines its degree of dependence, noting that if the order of integration of the series is equal to 1, it is possible for the efficiency market hypothesis to be satisfied provided the differenced process is uncorrelated. Moreover, it tests the hypothesis of mean reversion (orders of integration below 1 in prices) or alternatively, long memory returns (orders of integration above 1 in the log prices) by using long memory and fractional integration techniques. These are more general than the standard approaches based on integer degrees of differentiation, and provide much more flexibility in modelling the dynamics of the process. F vided the die09yvTf10.a9eferentiati .-iod o18.D

2. ÷ΠΤ2 1 Tfg memory and fractional integration

Given the parameterisation in (4) we can distinguish several cases depending on the value of d. Thus, if d = 0, $x_t = u_t$, x_t is said to be "short memory" or I(0), and if the observations are autocorrelated (i.e. AR) they are of a "weakly" form, in the sense that the values in the autocorrelations are decaying at an exponential rate; if d > 0, x_t is said to be "long memory", so named because of the strong association between observations far distant in time. If d belongs to the interval (0, 0.5) x_t is still covariance stationary, while d $\geq$ 0.5 implies nonstationarity. Finally, if d < 1, the series is mean reverting in the sense that the effects of shocks disappear in the long run, contrary to what happens if d $\geq$ 1 when they persist forever.

There exist several methods for estimating and testing the fractional differencing parameter d. Some of them are parametric while others are semiparametric and can be specified in the time or in the frequency domain. In this paper, we use a Whittle estimate of d in the frequency domain (Dahlhaus, 1989) along with a testing procedure, which is based on the Lagrange Multiplier (LM) principle and that also uses the Whittle function in the frequency domain. It tests the null hypothesis:

$$H_o: \; d \; = \; d_o, \qquad (5)$$

for any real value d_o, in a model given by the equation (4), where x_t can be the errors in a regression model of the form:

$$y_t \; = \; \beta^T z_t \; + \; x_t, \qquad t \; = \; 1, 2, ..., \qquad (6)$$

where y_t is the observed time series, β is a (kx1) vector of unknown coefficients and z_t is a set of deterministic terms that might include an intercept (i.e., $z_t = 1$), an intercept with a linear time trend ($z_t = (1, t)^T$), or any other type of deterministic processes. Robinson (1994) showed that, under certain very mild regularity conditions, the LM-based statistic $(\hat{r})$:

$$\hat{r} \; \rightarrow_d \; N(0, 1) \qquad \text{as} \qquad T \rightarrow \; \infty, \qquad (7)$$

where " $\rightarrow_d$ " stands for convergence in distribution, and this limit behaviour holds independently of the regressors z_t used in (6) and the specific model for the I(0) disturbances u_t in (4).

As in other standard large-sample testing situations, Wald and LR test statistics against fractional alternatives have the same null and limit theory as the LM test of Robinson (1994). Lobato and Velasco (2007) essentially employed such a Wald testing procedure, although it requires a consistent estimate of d; therefore the LM test of Robinson (1994) seems computationally more attractive. A semiparametric Whittle

As a first step we estimate a model of the form given by equations (4) and (6), with $z_t = (1,t)^T$, $t \geq 1$, 0, otherwise, i.e.,

$$y_t = \beta_0 + \beta_1 t + x_t, \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, ..., \qquad (8)$$

where y_t is the log-transformed price.

We report in Table 1 the estimates of d in (8) for the three standard cases of no regressors in the undifferenced regression (i.e., $\beta_0 = \beta_1 = 0$ in (8)), an intercept (β_0 unknown and $\beta_1 = 0$), and an intercept with a linear time trend (β_0 and β_1 unknown) along with the 95% confidence interval of the non-rejection values of d using Robinson (1994) parametric approach.

[Insert Table 1 about here]

The results are reported for the cases of both uncorrelated and autocorrelated errors. In the latter case, we assume first that u_t is an AR(1) process, but then also model the disturbances following the more general specification proposed by Bloomfield (1973). His is a non-parametric approach that approximates ARMA models with only a

Next we examine the volatility of the series measured as its absolute and squared returns.[1] Both series are displayed in Figure 2 along with their corresponding correlograms and periodograms. It can be seen that the sample autocorrelation values now decay very slowly, and the periodograms display large peaks at the zero frequency. This is clearly consistent with the I(d) process presented in Section 2 with a positive d.

[Insert Tables 2 and 3 about here]

Tables 2 and 3 provide the same information as Table 1 but for absolute and

and others. This method is essentially a local 'Whittle estimator' in the frequency domain, which uses a band of frequencies that degenerates to zero. The estimator is implicitly defined by:

$$\hat{d} = \arg\min_{d} \left(\log \overline{C(d)} - 2\,d\,\frac{1}{m}\sum_{s=1}^{m}\log \lambda_s \right), \qquad (9)$$

$$\overline{C(d)} = \frac{1}{m}\sum_{s=1}^{m} I(\lambda_s)\,\lambda_s^{2d}, \qquad \lambda_s = \frac{2\pi s}{T}, \qquad \frac{m}{T} \to 0,$$

where m is a bandwidth parameter, $I(\lambda_s)$ is the periodogram of the raw time series, x_t, given by:

$$I(\lambda_s) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} x_t\, e^{i\lambda_s t}\right|^2,$$

and d $\in$ (-0.5, 0.5). Under finiteness of the fourth moment and other mild conditions, Robinson (1995) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \to_d N(0, 1/4) \quad as\ T \to \infty,$$

where d_o is the true value of d. This estimator is robust to a certain degree of conditional heteroscedasticity (Robinson and Henry, 1999) and is more efficient than other more recent semi-parametric competitors.

[Insert Figure 3 and Table 4 about here]

Figure 3 displays the estimates of d for the return series and the absolute and squared returns, specifically the whole range of values of the bandwidth

As a final step we examine whether there are any anomalies related to the days of

4. Conclusions

In this paper we have examined the properties of the Ukranian stock market by estimating the order of integration of the PFTS series, daily, from January 9, 2007 until February 27, 2013. The main findings are the following. First, the log-prices series is highly persistent, with an order of integration significantly above 1, which implies that stock returns are characterised by long memory behaviour. Second, the same feature is detected in the absolute and squared returns which are used as a measure of volatility. Finally, the analysis by day of the week produces evidence of higher degrees of dependence on Mondays and Fridays than on the other days of the week.

References

Abadir, K.M., W. Distaso and L. Giraitis, 2007, Nonstationarity-extended local Whittle estimation, Journal of Econometrics 141, 1353-1384.

Baillie, R., T. Bollerslev and H. Mikkelsen, (1996), Fractionally integrated generalized autoregressive conditional heteroskedasticity, Journal of Econometrics 96, 3-30.

Bloomfield, P. (1973) An exponential model in the spectrum of a scalar time series, Biometrika 60, 217-226.

Brooks, C. and G. Persand (2001), Seasonality in Southeast Asian stock markets: Some new evidence on day –of –the week effects, Applied Economics Letters 8, 155 - 8.

Cross F. (1973) The behavior of stock prices on Fridays and Mondays, Financial Analyst Journal 29 (1973), 67-69.

Dahlhaus, R. 1989. Efficient parameter estimation for self-similar process. Annals of Statistics 17, 1749-1766.

Bollerslev, T. (1986), Generalized autoregressive conditional heteroskedasticity, Journal of Econometrics 31, 307-327.

Bollerslev, T. and H. Wright, (2000) High frecuency data, frequecy domain inference and volatility forecasting, Review of Economics and Statistics 83, 596-602.

Ding, Z., C.W.J. Granger, R.F. Engle (1993) A long memory property of stock markets and a new model, Journal of Empirical Finance 1, 83-106.

Engle, R.F. (1982) Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation, Econometrica 50, 987-1007.

French, K. (1980) Stock returns and the weekend effect, Journal of Financial Economics 8, 55-69.

Gibbons, M. and P. Hess (1981) , Day of the week effects and asset returns, Journal of Business 54, 579-596.

Gil-Alana, L.A. (2003) Fractional integration in the volatility of asset returns, European Review of Economics and Finance 2, 41-52.

Gil-Alana, LA. (2005) Long memory in daily absolute and squared returns in the Spanish stock market, Advances in Investment Analysis and Portfolio Management 1, 198-217.

Granger, C.W.J. and Z. Ding (1996), Varieties of long memory models, Journal of Econometrics 73, 61-78.

Jaffe, J.F. and R. Westerfield (1985), The week-end effect in common stock returns. The international evidence, Journal of Finance 40, 433-454.

Lobato, I. And N.E. Savin, (1998) Real and spurious long memory properties of stock market data, Journal of Business and Economic Statistics 16, 261-268.

Lobato, I. and C. Velasco (2007), Efficient Wald tests for fractional unit roots, Econometrica 75, 575-589.

McLeod, A.I. and K.W. Hipel (1978), "Preservation of the Rescaled Adjusted Range: A Reassessment of the Jurst Phenomenon", Water Resources Research, 14, 491-507.

Osborne, M.F.M. (1962) Periodic structure in the Brownian motion of the stock market, Operations Research 10. 345-379.

Phillips, P.C.B. and Shimotsu, K., 2004, Local Whittle estimation in nonstationary and unit root cases, Annals of Statistics 32, 656-692.

Phillips, P.C.B. and Shimotsu, K., 2005, Exact local Whittle estimation of fractional integration, Annals of Statistics 33, 1890-1933.

Robinson, P.M. 1994. Efficient tests of nonstationary hypotheses. Journal of the American Statistical Association 89, 1420-1437.

Robinson, P.M., 1995, Gaussian semi-parametric estimation of long range dependence, Annals of Statistics 23, 1630-1661.

Robinson, P.M. and M. Henry, 1999, Long and short memory conditional heteroskedasticity in estimating the memory in levels, Econometric Theory 15, 299-336.

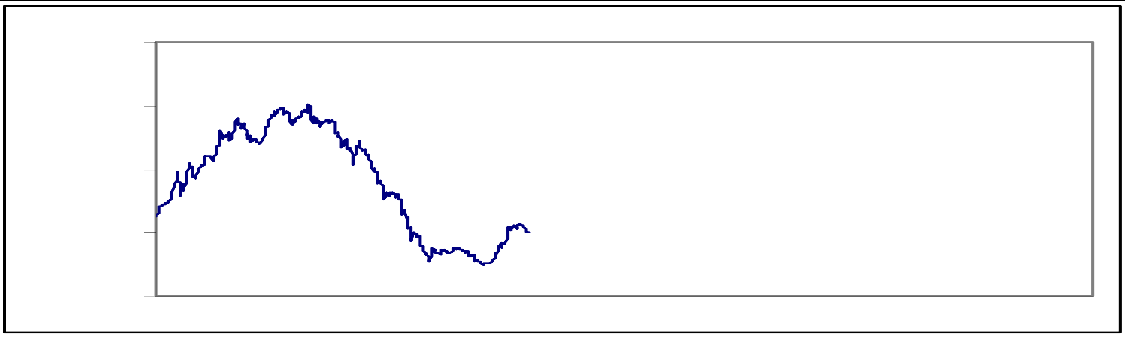
Solnik, B. and L. Bousquet (1990), Day of the week effect on the Paris Bourse, Journal of Banking and Finance 14, 461-469.

Velasco, C., 1999, Gaussian semiparametric estimation of nonstationary time series, Journal of Time Series Analysis 20, 87-127.

Velasco, C. and P.M. Robinson, 2000, Whitle pseudo maximum likelihood estimation for nonstationary time series, Journal of the American Statistical Association 95, 1229-1243.

Figure 1: Time series plots, correlograms and periodograms

i) PFTS stock market prices



ii) Stock market returns

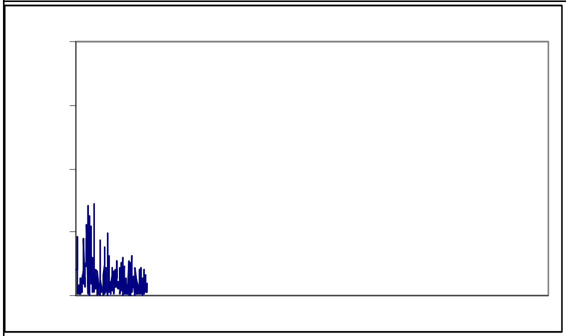
iii) Correlogram of the stock market returns*

iv) Periodogram of the tock market returns**

*: The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation.

**: The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$, j = 1, …, T/2.

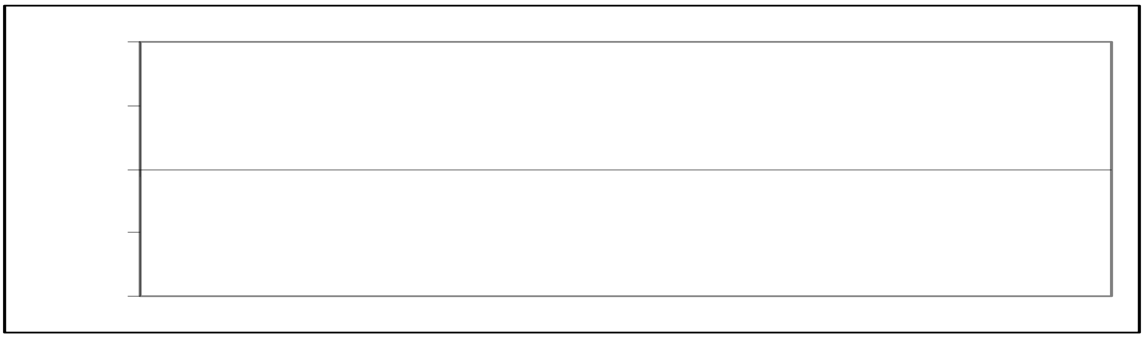
Figure 2: Absolute and squared returns, correlograms and periodograms

Absolute returns	Squared returns
	
Correlogram absolute returns*	Correlogram squared returns*
Periodogram absolute returns**	Periodogram squared returns**

*: The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation.
**: The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$, $j = 1, \ldots, T/2$.

Figure 3: Estimates of d based on the semiparametric approach of Robinson (1995)

i) Stock market returns



ii) Absolute returns

iii) Squared returns

The horizontal axis concerns the bandwidth parameter while the vertical one refers to the estimated value of d.

Table 1: Estimates of the fractional differencing parameter in the log of PFTS series

Table 5: Estimates of the fractional differencing parameter with white noise errors

	No regressors	An intercept	A linear time trend
Monday	1.017 (0.952, 1.100)	**1.187** **(1.124, 1.366)**	1.187 (1.124, 1.365)
Tuesday	1.016 (0.951, 1.099)	**1.144** **(1.085, 1.219)**	1.144 (1.085, 1.218)
Wednesday	1.013 (0.949, 1.096)	**1.135** **(1.077, 1.208)**	1.135 (1.077, 1.208)
Thursday	1.013 (0.948, 1.095)	**1.164** **(1.102, 1.244)**	1.164 (1.102, 1.243)
Friday	1.014 (0.949, 1.097)	**1.212** **(1.146, 1.296)**	1.212 (1.146, 1.295)

The values in parentheses give the 95% confidence band for the non-rejection values of d. In bold, the values corresponding to significant deterministic terms.

Table 6: Estimates of the fractional differencing parameter with AR(1) errors

	No regressors	An intercept	A linear time trend
Monday	1392 (1.280, 1.552)	**1.253** **(1.130, 1.413)**	1.252 (1.130, 1.408)
Tuesday	1.387 (1.266, 1.542)	**1.222** **(1.121, 1.353)**	1.221 (1.121, 1.350)
Wednesday	1.376 (1.258, 1.528)	**1.207** **(1.105, 1.327)**	1.206 (1.105, 1.324)
Thursday	1.375 (1.256, 1.526)	**1.174** **(1.069, 1.293)**	1.173 (1.069, 1.293)
Friday	1.384 (1.266, 1.537)	**1.228** **(1.095, 1.385)**	1.227 (1.095, 1.380)

The values in parentheses give the 95% confidence band for the non-rejection values of d. In bold, the values corresponding to significant deterministic terms.

Table 7: Estimates of the fractional differencing parameter with Bloomfield errors

	No regressors	An intercept	A linear time trend
Monday	1.012 (0.911, 1.147)	**1.242** **(1.123, 1.400)**	1.242 (1.123, 1.402)
Tuesday	1.002 (0.901, 1.147)	**1.231** **(1.111, 1.397)**	1.230 (1.111, 1.386)
Wednesday	1.003 (0.902, 1.046)	**1.213** **(1.091, 1.366)**	1.212 (1.091, 1.375)
Thursday	0.991 (0.906, 1.132)	**1.177** **(1.061, 1.321)**	1.177 (1.061, 1.319)
Friday	1.001 (0.894, 1.131)	**1.219** **(1.102, 1.380)**	1.218 (1.101, 1.377)

The values in parentheses give the 95% confidence band for the non-rejection values of d. In bold, the values corresponding to significant deterministic terms.

Table 8: Semiparametric estimates of d: Robinson (1995) and Abadir et al. (2007)

Bandwith nb.	Monday	Tuesday	Wednesday	Thursday	Friday
5	0.130	0.128	0.138	0.154	0.138
10	0.500	0.500	0.500	0.500	0.500
15	0.101	0.089	0.093	0.106	0.105
18***	0.096	0.093	0.096	0.101	0.097
20	0.084	0.093	0.100	0.095	0.085
25	0.181	0.191	0.100	0.200	0.189
30	0.186	0.182	0.191	0.198	0.192

***: Bandwidth number corresponding to $(T)^{0.5}$.

Table 9: Estimates of the fractional diff