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AFRICAN GROWTH, NON-LINEARITIES AND STRONG DEPENDENCE:

presence of breaks that are not taken into account. Further, changes can occur smoothly rather than suddenlnlnlnl

Fosu (1992b) investigated the effect of export instability on GDP growth in Africa, and found that these are particularly significant in the case of sub-Saharan Africa. Karikari (1995) examined the role of the government in the growth of a developing nation, using data for Ghana from 1963 to 1984. He concluded that the impact of government on economic growth was negative. Savvides (1995) investigated the factors that explain the differences in per capita growth across Africa, and concluded that these are: initial conditions, investment, economic growth, trade orientation, inflation, financial development and the growth of the government sector. Easterly and worldwide, which has a large negative effect on productivity growth in the South African manufacturing industry.

3. Methodology

As a first step, we carry out standard unit root tests, specifically the Augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1979), as well as its generalization, i.e. the GLS specification (Elliot el al., ERS, 1996), and the Kwiatkowski et al. (KPSS, 1992) test for the null of stationarity against the alternative of a unit root.

We then consider the following non-linear model:

$$y_t = \sum_{i=0}^{m} \theta_i P_{iT}(t) + x_t, \qquad t = 1, 2, \ldots, \qquad (1)$$

with m indicating the order of the Chebyshev polynomial, and x_t following an I(d) process of the form

$$(1 - L)^d x_t = u_t, \qquad t = 0, 1, \ldots,$$

An issue that immediately arises here is how to determine the optimal value of *m*. As argued in Cuestas and Gil-Alana (2012), if one combines (1) and (2) in a single equation, standard *t*-statistics will remain valid with the error term being *I(0)* by definition. The choice of *m* will then depend on the significance of the Chebyshev coefficients. Note that the model combining (1) and (2) becomes linear and *d* can be estimated parametrically or tested as in Robinson (1994), Demetrescu, Kuzin and Hassler (2008) and others (see Cuestas and Gil-Alana, 2012).

The method proposed here is a slight modification of (fic)-4(a)-5(ti)29(p1 85.f6BT/F3 12 Tf1 5 which can also be expressed as in Robinson (1994) ($P_T^*(t) \;\; z_t^*$), and then, using OLS/GLS methods, under the null hypothesis (4), the residuals are

$$\hat{u}_t \quad y_t^* \quad \sum_{i=0}^{m} \hat{\beta}_i \, P_{iT}^*(t);$$

with

and $P_T^*(t)$ as the ($mx1$) vector of transformed Chebyshev polynomials. Using the above residuals $\hat{u}_t$, we estimate the variance,

$$\hat{\sigma}^2(\tau) \quad \frac{2}{T} \sum_{j=1}^{T} g(\lambda_j;\hat{\tau})^{-1} I_{\hat{u}}(\lambda_j); \qquad \lambda_j \quad 2\pi j/T, \tag{7}$$

where $I_{\hat{u}}(\lambda_j)$ is the periodogram of $\hat{u}_t$; g is a function related to the spectral density of u_t (i.e., s.d.f.(u_t) = $(\sigma^2/2\pi)g(\lambda_j;\tau)$); and the nuisance parameter τ is estimated, for example, by $\hat{\tau} \;\; \arg\min_{\tau \in T^*} \sigma^2(\tau)$, where T^* is a suitable subset of the R^q Euclidean space.[2]

The test statistic (based on Robinson (1994)) for testing H_o (4) in (1) and (2) uses the Lagrange Multiplier (LM) principle, and is given by

$$\hat{R} \quad \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a}, \tag{8}$$

where T is the sample size, and

$$\text{———} \qquad \hat{\tau}) \quad (\quad),$$

$$\hat{A} \quad \frac{2}{T} \quad \sum_j^{*} \psi(\lambda_j)\,\psi(\lambda_j)' \quad \sum_j^{*} \psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \quad \sum_j^{*} \hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)' \quad \sum_j^{*} \hat{\varepsilon}(\lambda_j)\,\psi(\lambda_j)'$$

Alternative methods for estimating the variance, e.g., non-parametric ones, could also be used. Here we take the same approach as in Robinson (1994).

with

$$(\ _j) \quad \mathrm{Re}\ \frac{}{d}\log \quad (e^{i\ \ j};\, d)\ , \quad \text{and}\ \hat{\ }(\ _{\mathrm{j}}) \quad \text{—}\log g(\ _{\mathrm{j}};\ )\Big| \quad \hat{\ },$$

and the sum over * above refers to all the bounded discrete frequencies in the spectrum. Under very mild regularity conditions,[3] it can be shown that, as in Robinson (1994)

$$\hat{R} \quad _d \quad {}^{2}_{1} \qquad as \quad T \qquad , \qquad\qquad (9)$$

and, based on Gaussianity of u_t, one can also show the Pitman efficiency of the test against local departures from the null. In other words, if one considers local alternatives of the form:

$$H_a:\ d \qquad d_o \qquad T^{\ 1/2},$$

Table 1 provides a list of the countries with the corresponding sample periods, the longest being those starting in 1950 for the Congo Democratic Republic, Ethiopia, Morocco, Nigeria, South Africa and Uganda. The start date is 1954 for Zimbabwe, 1955 for Zambia and Ghana, 1960 for Algeria, Botswana, Burundi, Central African Rep., Chad, Congo Republic, Cape Verde, Equatorial Guinea, Gambia, Guinea Bissau, Mali, Mauritania, Mozambique, Namibia and Niger, 1961 for Sierra Leone and Tunisia, 1970 for Angola and Somalia. The end date is 2010 in all cases.

[Insert Tables 2 and 3 about here]

The unit root test results (ADF, KPSS and ERS) reported in Tables 2 (in levels) and 3 (in first differences) suggest that the level series are I(1), whilst the GDP growth rates are I(0) in all cases. However, it is well known that such tests have very low power if the DGP is characterised by fractionally integration (see, Diebold and Rudebusch, 1991; Hassler and Wolters, 1994; Lee and Schmidt, 1996; and more recently Ben Nasr *et al*., 2014); on the other hand, fractional integration may be a spurious phenomenon caused by the presence of non-linearities and structural breaks in the data that have not been taken into account.[4] For these reasons, next we allow for non-linear trends in the context of fractional integration, and consider the following model,

$$y_t \qquad {}_{i \;\; 0}^{\;\; m} \; {}_i P_{iT}(t) \qquad x_t , \qquad (1 \qquad L)^d x_t \qquad u_t ,$$

95% confidence bands showing the values of d for which the null hypothesis (4) cannot be rejected. The remaining columns display the estimated coefficients along with their corresponding t-values.

[Insert Table 4 about here]

For the Central African Republic, Niger, Sierra Leone and Somalia there is no evidence of non-linearities, since the two coefficients on the non-linear terms (i.e., θ_2 and θ_3) are statistically insignificant. Further, the order of integration varies considerably across these countries: for the Central African Republic and Somalia, the estimated value of d is significantly smaller than 1 (0.37 and 0.49 respectively), which implies in both cases mean-reverting behaviour; for Niger the estimate of d is below 1, but the unit root null hypothesis cannot be rejected; and for Sierra Leone the estimated value of d is 1.32 and the hypothesis of d = 1 is decisively rejected in favour of d > 1.

The countries exhibiting a large degree of non-linearity are those for which all four coefficients are statistically significant, namely Cabo Verde, Equatorial Guinea, Gambia, Mauritania, Mozambique and Uganda. In four of them (Cabo Verde, Equatorial Guinea, Mozambique and Uganda) the unit root null (i.e., d = 1) cannot be rejected, while for the remaining two (Gambia and Mauritania) the null of mean reversion (i.e., d < 1) cannot be rejected.

In between, there are some cases with at least one of the two non-linear coefficients being statistically significant. Specifically, a significant θ_3 is found for Algeria, Ethiopia, Gambia, Morocco, Nigeria, Namibia, South Africa, Tunisia and Zambia, and a significant θ_2-coefficient for Botswana, Burundi, Chad, Congo Democratic Republic, Congo Republic, Guinea Bissau and Mali. For this group of countries, mean reversion (d < 1) is found in Algeria, Botswana, Guinea Bissau, Malia, Namibia and Tunisia, whilst the unit root null cannot be rejected in Angola, Burundi, Chad, Congo Democratic Republic, Congo, Ethiopia, Ghana, Morocco, Nigeria, South Africa, Zambia and Zimbabwe. Therefore, we can conclude by saying that there is some evidence of non-

Sierra Leone and Somalia. For the remaining countries strong evidence of non-linearities is obtained for Cabo Verde, Equatorial Guinea, Gambia, Mauritania, Mozambique and Uganda, followed by Algeria, Ethiopia, Gambia, Morocco, Nigeria, Namibia, South Africa, Tunisia and Zambia (where θ_3 is statistically significant), and for Botswana, Burundi, Chad, Congo Democratic Republic, Congo Republic, Guinea Bissau and Mali (with a significant θ_2-coefficient).

Heterogeneity across countries is another feature of our results, mean-reversion, unit root behaviour and orders of integration significantly higher than 1 being found in different cases. Overall, the evidence presented in this study confirms the importance of taking into account non-linearities when modelling GDP per capita in countries such as the African ones where various types of conflicts have disrupted economic growth at different stages.

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Table 2: Unit root test results (levels)

Country	ADF	KPSS	ERS
	Intercept		

Table 4: Estimated coefficients in a model with m = 3

Country	d (95 interval)	θ_0	θ_1	θ_2	θ_3
Angola	1.16 (0.93, 1.45)	**2355.81 (1.87)**	-315.19 (-0.41)	**555.24 (1.73)**	-240.75 (-1.20)
Algeria	0.60 (0.33, 0.93)	**4991.13 (16.39)**	**-663.64 (-3.82)**	-53.06 (0.41)	**-265.79 (-2.57)**
Botswana	0.56 (0.21, 0.98)	**4882.13 (14.67)**	**-3103.37 (-16.13)**	**306.28 (2.07)**	-192.36 (-1.60)
Burundi	0.88 (0.61, 1.25)	**468.54 (7.56)**	-13.64 (-0.38)	**-75.24 (-3.72)**	-0.67 (-0.04)
Central African Rep.	0.37 (0.11, 0.72)	**760.41 (43.43)**	**-163.42 (-14.19)**	-1.65 (-0.16)	-7.19 (-0.83)
Chad	0.97 (0.65, 1.40)	**838.6281 (3.41)**	-78.58 (-0.55)	**122.42 (1.65)**	-62.39 (-1.24)

Tunisia	0.58	(0.29,	0.95)	**3940.28**

Tunisia	0.99 (0.85, 1.19)	**3507.53 (7.16)**	**-1230.76 (-4.14)**	28.65 (0.19)
Uganda	1.21 (1.07, 1.43)	**518.16 (2.09)**	4.23 (0.02)	84.58. (1.31)

Table 6: Estimated coefficients in a model with m = 1

Country	d (95% interval)	θ_0	θ_1
Angola	1.25 (1.11, 1.47)	**2376.49 (1.88)**	-18.71 (-0.02)
Algeria	0.77 (0.59, 1.03)	**4949.06 (10.36)**	**-743-33 (-2.45)**
Botswana	0.75 (0.56, 1.06)	**4102.00 (9.12)**	**-3137.99 (-8.88)**
Burundi	1.14 (0.99, 1.39)	**372.66 (2.81)**	-14.54 (-0.16)
Central African Rep.	0.37 (0.12, 0.73)	**758.57 (44.51)**	**161.72 (14.15)**
Chad	1.10 (0.91, 1.42)	**879.58 (2.64)**	-48.72 (-0.21)
Congo Dem. Rep.	1.03 (0.87, 1.23)	231.16 (1.21)	201.54 (1.54)
Congo Rep.	1.15 (0.93, 1.50)	**1481.14 (1.99)**	-350.40 (-0.68)
Cabo Verde	1.27 (1.15, 1.42)	**1431.93 (1.94)**	-413.11 (-0.80)
Equatorial Guinea	1.37 (1.23, 1.50)	5259.44 (0.69)	-3262.64 (-0.61)
Ethiopia	1.09 (0.97, 1.22)	**293.05 (2.31)**	-7.21 (-0.08)
Gambia	0.80 (0.61, 1.09)	**1126.75 (10.94)**	-8.54 (-0.12)
Ghana	1.08 (0.93, 1.27)	**1205.25 (3.20)**	-50.48 (-0.19)
Guinea Bissau	0.82 (0.67 1.04)	**764.22 (5.09)**	18.35 (0.18)
Mali	0.84 (0.70, 1.05)	**763.30 (11.17)**	**-153.92 (-3.60)**
Mauritania	0.90 (0.77, 1.07)	**1018.59 (3.56)**	-271.53 (-1.44)
Morocco	0.99 (0.86, 1.15)	**1913.43 (5.21)**	**-768.91 (-3.08)**
Mozambique	1.30 (1.21, 1.44)	**295.49 (1.82)**	7.96 (0.07)
Namibia	0.96 (0.82, 1.15)	**3104.69 (4.93)**	-223.46 (-0.53)
Niger	0.83 (0.58, 1.13)	**648.59 (7.50)**	**167.50 (2.99)**
Nigeria	1.19 (0.99, 1.54)	955.76 (1.28)	110.32 (0.21)
South Africa	1.27 (1.14, 1.51)	**3911.36 (3.05)**	-461.12 (-0.51)
Sierra Leone	1.38 (1.16, 1.81)	75.75 (0.13)	228.14 (0.56)
Somalia	0.49 (0.17, 0.92)	**606.23 (18.40)**	**124.54 (6.05)**

Table 7: Order of integration of each series according to the selected models

Country	m = 0	m = 1	m = 2	m = 3
Angola	1.25 (1.09, 1.49)	xxx		