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TRENDS AND CYCLES IN MACRO SERIES:

THE CASE OF US REAL GDP

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Abstract

In this paper we propose a new modelling framework for the analysis of macro series that includes both stochastic trends and stochastic cycles in addition to deterministic terms such as linear and non-linear trends. We examine four US macro series, namely annual and quarterly real GDP and GDP per capita. The results indicate that the behaviour of US GDP can be captured accurately by a model incorporating both stochastic trends and stochastic cycles that allows for some degree of persistence in the data. Both appear to be mean-reverting, although the stochastic trend is nonstationary whilst the cyclical component is stationary, with cycles repeating themselves every 6 – 10 years.

Keywords: GDP; GDP per capita; trends; cycles; long memory; fractional integration

JEL Classification: C22, E32

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1. Introduction

In this paper we put forward a new modelling framework for macro series that allows for two singularities (or poles) in the spectral density function, one corresponding to the long-run or zero frequency (i.e. to the long-run evolution of the series), the other to a non-zero frequency (and related to a cyclical pattern

2. Literature Review

GDP, whether nominal, real or per capita, is typically a non-stationary variable in most developed countries. For many years, the standard modelling approach was to use deterministic functions of time, usually of a linear form, as in the following specification:

$$y_t = \alpha + \beta t + x_t, \quad t = 1, 2, ..., \qquad (1)$$

where $\{y_t, t = 1, 2, …, T\}$ is the observed (GDP) series, α and β are the coefficients on an intercept and a linear time trend respectively, and x_t is assumed to be covariance stationary, usually of the ARMA form, to capture short-run and cyclical patterns in the data. Therefore, the process followed by x_t can be represented as

$$\phi(L) x_t = \theta(L) \varepsilon_t, \quad t = 1, 2, ..., \qquad (2)$$

where $\phi(L)$ and $\theta(L)$ stand for the AR and MA components of the series respectively. This modelling framework was dominant in the literature until the publication of a very influential paper by Nelson and Plosser (1982), who examined fourteen US macroeconomic series and by applying the tests developed by Fuller (1976) and Dickey and Fuller (1979) found evidence of unit roots and came to the conclusions that the behaviour of these variables except one could be better described in terms of stochastic trends, that is, as in the following model including an intercept:

$$y_t = \alpha + y_{t-1} + x_t, \quad t = 1, 2, ..., \qquad (3)$$

where x_t is I(0) and can be represented as in (2).[1] This model has been widely employed in the macro literature and in the last twenty years many additional unit root tests have been developed (Phillips and Perron, 1988; Elliot et al., 1996; Ng and Perron, 2001; etc.). These two specifications, i.e. the deterministic trend model as in (1) and the

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[1] For our purposes we define an I(0) process as a covariance stationary process, i.e. one for which the infinite sum of the autocovariances is finite. Alternatively, in the frequency domain, it can be defined as a process with a spectral density function that is positive and finite at all frequencies in the spectrum.

stochastic trend model as in (3), can coexist within the same framework if x_t in (1) contains a unit root, the main difference between the two models being the treatment of shocks, which have transitory effects in the case of (1) but permanent ones in the case of (3). However, a process may display nonstationary, persistent behaviour but still be mean-reverting as in the I(d) models with a differencing parameter d lying in the interval [0.5, 1). In such models, x_t is specified as

$$1, 2, \ldots, \qquad (4)$$

where d can be any real value and u_t is I(0) (defined as in footnote 1). Variants of this model have been used to analyse the behaviour of GDP in various countries (see, e.g., Michelacci and Zaffaroni, 2000, Mayoral, 2006, Gil-Alana, 2010, Caporale and Gil-Alana, 2013, Caporale and Skare, 2014).

Cyclicality is another important feature of GDP series. There exists a large literature using different methods such as time-varying transition probabilities (TVTP) Markov-switching regime models (see, e.g., Simpson et al., 2001), band pass filters (Christiano and Fitzgerald, 1999), etc. A similar approach to equation (4) can also be used to allow stochastic cyclical processes to be fractional as in the following model,

$$(1 \quad 2\cos \quad L \quad L^2)^d x_t \quad u_t, \quad t \quad 1, 2, \ldots, \qquad (5)$$

with

$$(1 \quad 2 \quad L \quad L^2) \quad ^d \quad _{j \quad 0} \; C_{j,d}(\quad) L^j ,$$

where L stands for

Gray et al. (1989, 1994) showed that x_t in (5) is (covariance) stationary if $d < 0.5$ for $|\mu = \cos w_r| < 1$ and if $d < 0.25$ for $|\mu| = 1$. This process implies the existence of a pole or singularity at a non-zero frequency which corresponds to the cyclical pattern. Special cases of this model were analysed by Athola and Tiao (1987) and Bierens (2001) setting $d = 1$, and by Gil-Alana (2001), DePenya and Gil-Alana (2006) and

Robinson (1994) had previously proposed a general testing framework that

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$H_0 : z_t \; d_{X_t}, \; t \; d_0,$

Figure 1 displays the four series, all of which exhibit an upward trend suggesting non-stationary behaviour. This is confirmed by their correlograms (Figure 2) and the periodograms (Figure 3), the former decaying slowly and the latter exhibiting their highest values at the smallest frequencies. Figure 4 displays the same four series in first differences, with the corresponding correlograms and periodograms (displayed in Figures 5 and 6 respectively) providing evidence of cyclical patterns.

[Insert Figures 4 - 6 about here]

We start by considering a linear model with a time trend allowing for unit roots and fractional degrees of integration, specifically:

$$y_t \;=\; \alpha \;+\; \beta\, t \;+\; x_t, \qquad (1 - B)^d x_t \;=\; u_t, \qquad t = 1, 2, ..., \qquad (11)$$

where the errors are assumed to follow in turn a white noise and an autocorrelated process. However, instead of imposing a parametric ARMA structure on u_t, we employ a non-parametric method due to Bloomfield (1973) such that the error term is specified exclusively in terms of its spectral density function, which is given by

$$f_u(\lambda; \tau) \;=\; \frac{\sigma^2}{2\pi} \exp\left(2 \sum_{r=1}^{m} \tau_r \cos(\lambda r) \right),$$

[Insert Tables 1 and 2 about here]

In the white noise case the time trend is significant in all cases except annual GDP per capita, and the estimates of d are significantly above 1, ranging from 1.31 (quarterly GDP) to 1.45 (annual GDP per capita). When allowing for (weak) autocorrelation as specified by Bloomfield (1973), the time trend is significant in all four cases, and the estimated values of d are still significantly above 1 but smaller.

Given the significance of the time trend in most cases, next we investigate whether it might be non-linear by using an approach based on Chebyshev polynomials in time that has been shown to perform well in the context of the tests of Robinson (1994) for fractional integration (Cuestas and Gil-Alana, 2016). Thus, we replace the first (linear) equation in (11) with:

(13)

with m indicating the order of the Chebyshev polynomial $P_{i,T}(t)$ defined as:

$$P_{0,T}(t) \quad 1,$$

$$P_{i,T}(t) \quad \sqrt{2}\cos \, i \quad (t \quad 0.5)/T \,, \qquad t \quad 1, 2, \ldots, T; \quad i \quad 1, 2, \ldots \qquad (14)$$

(see Hamming (1973) and Smyth (1998) for a detailed description of these polynomials). Bierens (1997) and Tomasevic et al. (2009) argue that it is possible to approximate highly non-linear trends with polynomials of a rather low degree. This model includes the previous one noting that if $m = 0$ it contains an intercept, if $m = 1$ it includes a linear trend, and if $m > 1$ it becomes non-linear - the higher m is the less linear the approximated deterministic component becomes. Combining (13) with the second equation in (11) yields a linear model that can be estimated using least squares (see Cuestas and Gil-Alana, 2016).

[Insert Table 3 about here]

systematically higher than d_2, which indicates that the long-run frequency is relatively more important than the cyclical one. Specifically, d_1 ranges between 0.55 (annual real GDP, original data) to 1.24 (quarterly real GDP per capita, demeaned data), while d_2 oscillates around 0, being significantly positive for the original series at the annual frequency as well as for both annual and quarterly real GDP per capita in the case of the demeaned series.

Table 5 displays the results under the assumption of AR(1) errors. In this case the estimated value of r is 10 for the four annual series, whilst it is 7 and 10 respectively for quarterly real GDP and real GDP per capita. Moreover, the estimated value of d_1 is much lower than in the previous case, and is not statistically different from zero for some of the original series. This might be a consequence of the competition with the AR(1) parameter in describing the degree of persistence in the long run structure of the data. For the demeaned series the values of d_1 are significant but smaller than those reported in Table 4. Besides, d_2 is now statistically significant in all cases, which implies the presence of a cyclical pattern.

Finally, Table 6 displays the results under the assumption that the error term follows the nonthe demeaned data; for quarterly real GDP per capita framework with the aim of distinguishing between different types of shocks affecting trends and cycles separately while still allowing for a flexible degree of persistence.

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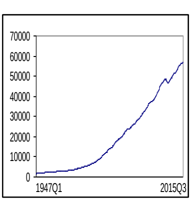

70000
60000
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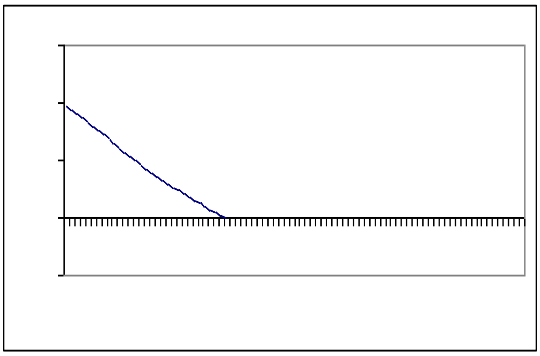
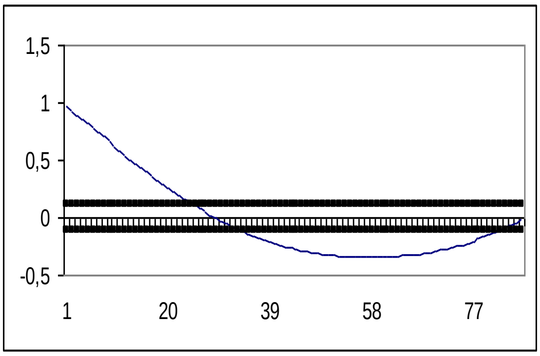
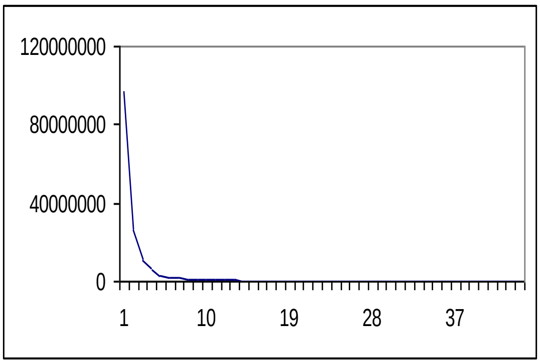
Figure 2: Correlograms of the original time series

Annual Real GDP	Quarterly Real GDP
	
Annual Real GDP per capita	**Quarterly Real GDP per capita**
	

The thick lines refer to the 95% confidence bands for the null hypothesis of no autocorrelation.

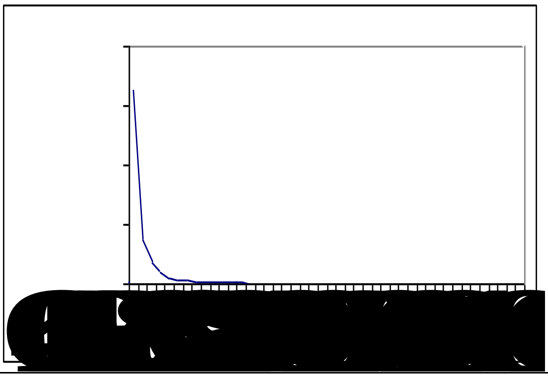
Figure 3: Periodograms of the original time series

Annual Real GDP



Quarterly Real GDP

Annual Real GDP per capita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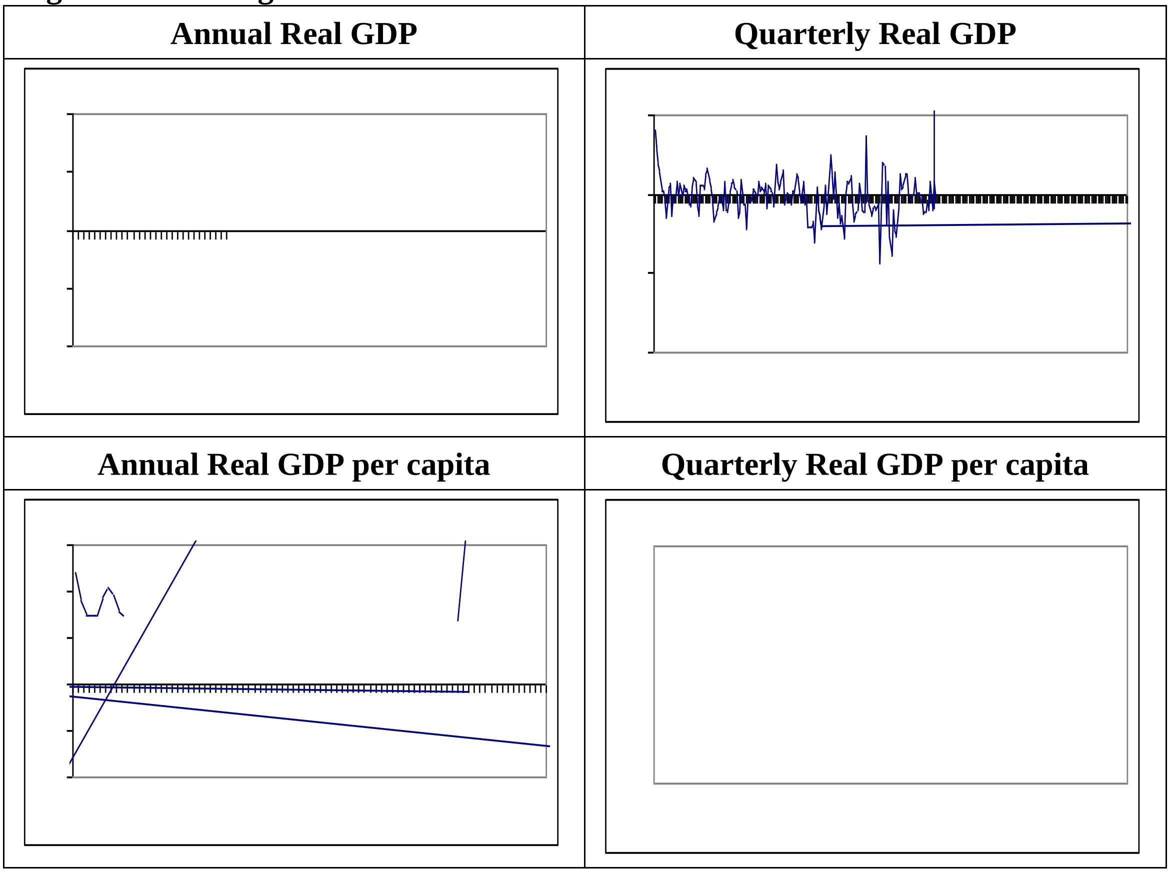
Quarterly Real GDP per capita

The horizontal axis refers to the discrete Fourier frequencies λj = 2πj/T, j = 1, 2, …T/2.

Figure 4: First differenced time series

Annual Real GDP **Quarterly Real GDP**

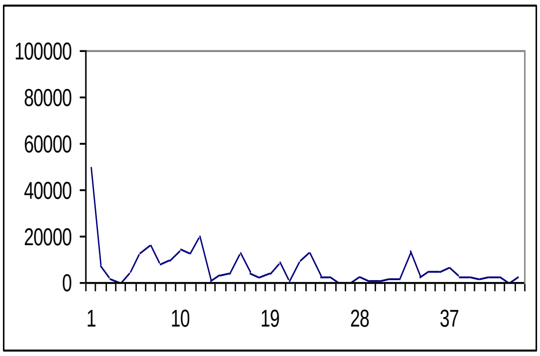
Figure 5: Correlograms of the first differenced time series



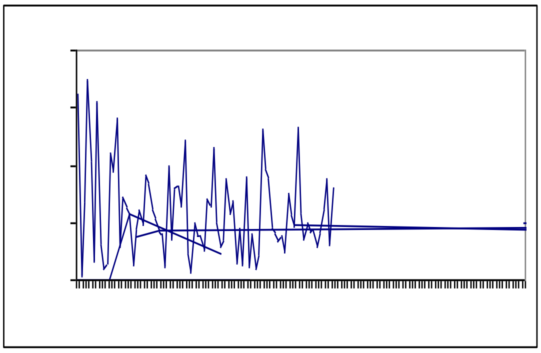
The thick lines refer to the 95% confidence bands for the null hypothesis of no autocorrelation.

Figure 6: Periodograms of the first differecend time series

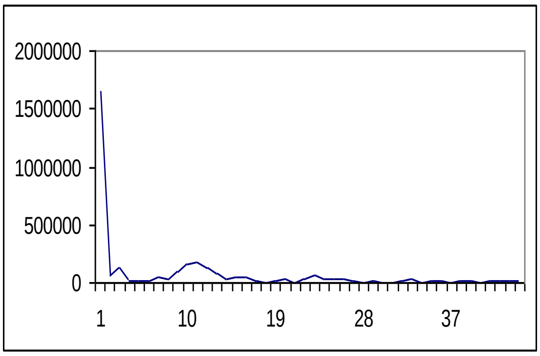
Annual Real GDP



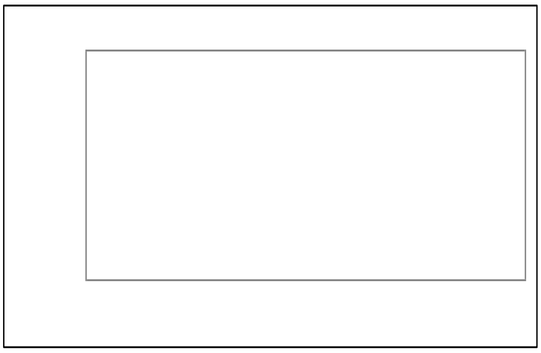
Quarterly Real GDP



Annual Real GDP per capita



Quarterly Real GDP per capita



The horizontal axis refers to the discrete Fourier frequencies $\lambda j = 2\pi j/T$, j = 1, 2, …T/2.

Table 1: Estimated values of d with white noise errors

Series	No terms	An intercept	A linear trend
Annual real GDP	1.22 (1.11, 1.42)	1.31 (1.18, 1.58)	**1.36 (1.23, 1.58)**
Annual real GDP per cap	1.45 (1.34, 1.65)	**1.45 (1.35, 1.68)**	1.49 (1.39, 1.68)
Quarterly real GDP	1.09 (1.02, 1.18)	1.30 (1.22, 1.41)	**1.31 (1.24, 1.42)**
Quarterly real GDP per cap	1.33 (1.26, 1.42)	1.38 (1.31, 1.48)	**1.40 (1.34, 1.49)**

Table 4: Estimated coefficients with